

# Deterministic Discrete Modeling

## Formal Semantics of Firewalls in Isabelle/HOL

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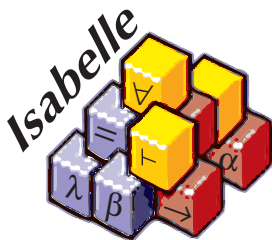
# Agenda

- 1 Introduction to Isabelle/HOL
  - PL, FOL, HOL
  - Isabelle/HOL
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- 3 Modeling Firewalls
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  - Syntax
  - Semantics
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Slides: 63

## About Isabelle/HOL

- ▶ Generic proof assistant
- ▶ For
  - ▶ interactive theorem proving
  - ▶ in higher-order logics (HOL)



<http://isabelle.in.tum.de/>

```
$ wget http://isabelle.in.tum.de/dist/Isabelle2013-2_linux.tar.gz
$ sha1sum Isabelle2013-2_linux.tar.gz
cb8dca7fdc9090b7640121ca6b575a714a62a492 Isabelle2013-2_linux.tar.gz
$ tar -xzf Isabelle2013-2_linux.tar.gz
$ ./Isabelle2013-2/bin/isabelle jedit &
```

# About Isabelle/HOL

## Note

Note: The theory files are relevant for the exam! We will soon leave the slide set and interactively work with Isabelle/HOL.

### About this Chapter

- ▶ Please make sure you have a laptop and Isabelle/HOL with you (alternatively, find a fellow student who does)
- ▶ There is a [firewall2.thy](#) on the website.

**This is the accompanying exercise.**

It contains several TODOs. Try to solve them after each lecture.

## Propositional Logic, First Order Logic

- ▶ Propositional Logic

$$(a \longrightarrow b) \vee (b \longrightarrow a)$$

- ▶ First Order Logic (FOL)

FOL = Propositional Logic + Quantifiers

- ▶ Example FOL

$$\forall a. \exists b. (a \longrightarrow b) \vee (b \longrightarrow a)$$

$$(a \longrightarrow b) \vee (b \longrightarrow a)$$

## HOL – Higher-Order Logic

- ▶ Higher-Order Logic (HOL)

HOL = Functional Programming + Logic

- ▶ Example FOL

$$(a \longrightarrow b) \vee (b \longrightarrow a)$$

- ▶ Example Lambda Calculus

$$(\lambda x. x + 4) 38$$

- ▶ Example Transitive Closure of a Relation  $R$

$$(a, b) \in R \wedge (b, c) \in R \longrightarrow (a, c) \in R^+$$

## Isabelle/HOL

- ▶ Isabelle/HOL: A Proof Assistant for Higher-Order Logic

We model and reason in HOL

- ▶ Example FOL

$$(a \longrightarrow b) \vee (b \longrightarrow a)$$

- ▶ In Isabelle/HOL

lemma “ $(a \longrightarrow b) \vee (b \longrightarrow a)$ ” by auto

- ▶ Lambda Calculus

$$(\lambda x. x + 4) 38$$

- ▶ In Isabelle/HOL

lemma “ $(\lambda x. x + 4) 38 = 42$ ” by simp



## Isabelle/HOL Examples

### ▶ Example

lemma “ $(a \longrightarrow b) \vee (b \longrightarrow a)$ ” by auto

### ▶ Terms are written in “”

### ▶ lemma starts a lemma

### ▶ it can be proven automatically by auto

### ▶ other automated proof methods

#### ▶ simp – the simplifier

#### ▶ auto – simplification, logic, sets, may return unprovable goals

#### ▶ blast – complete for FOL

#### ▶ ...

lemma “ $(a \longrightarrow b) \vee (b \longrightarrow a)$ ” by simp

lemma “ $(a \longrightarrow b) \vee (b \longrightarrow a)$ ” by blast

lemma “ $(a \longrightarrow b) \vee (b \longrightarrow a)$ ” by fastforce

## Isabelle/HOL Examples

- ▶ About the transitive closure

lemma “ $(a, b) \in R \wedge (b, c) \in R \implies (a, c) \in R^+$ ” by auto

- ▶ About the reflexive transitive closure

lemma “ $(a, a) \in R^*$ ” by simp

- ▶ FOL is modeled in HOL

- ▶ HOL implication:  $\implies$
- ▶ FOL implication:  $\longrightarrow$

lemma “ $(a, b) \in R \wedge (b, c) \in R \longrightarrow (a, c) \in R^+$ ” by auto

## Isabelle/HOL

- ▶ Implications associate to the right

$$A \implies B \implies C \text{ means } A \implies (B \implies C)$$

- ▶ This is equal to

$$A \wedge B \implies C$$

- ▶ or

lemma assumes  $A$  and  $B$  shows  $C$

- ▶ Homework: prove

$$(a \longrightarrow b \longrightarrow c) \longleftrightarrow (a \wedge b \longrightarrow c)$$

on paper and in Isabelle/HOL

## Isabelle/HOL Notation

- ▶ FOL all quantifier:  $\forall$
- ▶ HOL all quantifier:  $\bigwedge$
- ▶ Example
  - ▶ lemma ‘‘ $\forall x. x$ ’’
    - note: obviously wrong, counterexample  $x = \text{False}$
  - ▶ apply(rule)
    - produces subgoal  $\bigwedge x. x$
    - From FOL to HOL
- ▶ “free” variable (can be instantiated):  $?x$
- ▶ Example
  - ▶ lemma  $x$ : ‘‘ $a = a$ ’’ by simp
  - ▶ thm  $x$  gives you  $?a = ?a$
  - ▶ That is, you can instantiate lemma  $x$  for arbitrary  $?a$
  - ▶ Example: thm  $a$ [of "foo"] is  $\text{foo} = \text{foo}$
- ▶ Concluding Example: Inspect allI

# Types & Functional Programming

*Does the set of all sets contain itself?*

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- ▶ If not: It is obviously not the set of all sets! ⚡

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- ▶ If not: It is obviously not the set of all sets! ⚡
- ▶ If it does: Let  $A$  be the subset that contains all sets that do not contain themselves.
  - ▶ Is  $A \in A$  ?
  - ▶ If not:  $A$  is obviously not the set of all sets that do not contain themselves! ⚡
  - ▶ If it does:  $A$  contains a set that contains itself! ⚡



## Types

- ▶ Why is HOL typed?
- ▶ An example of untyped mathematics:

*Does the set of all sets contain itself?*

- ▶ If not: It is obviously not the set of all sets! ⚡
- ▶ If it does: Let  $A$  be the subset that contains all sets that do not contain themselves.
  - ▶ Is  $A \in A$  ?
  - ▶ If not:  $A$  is obviously not the set of all sets that do not contain themselves! ⚡
  - ▶ If it does:  $A$  contains a set that contains itself! ⚡

## Types

- ▶ Typed sets
  - ▶ nat set e.g.,  $1 \in \{1, 2\}$
  - ▶ nat set set e.g.,  $\{1, 2\} \in \{\{1, 2\}, \{\}\}$
  - ▶ nat set set set e.g.,  $\{\{1, 2\}, \{\}\} \in \{\{\{1, 2\}, \{\}\}, \{\{1, 2, 3, 4\}\}\}$
- ▶  $\in$  is of type  $\underbrace{\text{nat}}_1 \underbrace{\in}_{\in} \underbrace{\text{nat set}}_{\{1,2\}} \Rightarrow \underbrace{\text{bool}}_{\text{is true}}$
- ▶ The concept of *set of all sets* is not well-typed
- ▶  $\mathcal{M} = \text{the set of all sets}$
- ▶ What is the type of  $\mathcal{M}$ ?
  - ▶ There is no (finite) type for  $\mathcal{M}$ !
- ▶ Even if there were a type for  $\mathcal{M}$ ,  $\mathcal{M} \in \mathcal{M}$  is an obvious type error
- ▶ HOL is typed!

## Polymorphic Types

- ▶ Arbitrary types  $'a, 'b, \dots$
- ▶  $\in$  is of type  $'a \Rightarrow 'a \text{ set} \Rightarrow \text{bool}$
- ▶ For example
  - ▶  $\text{False} \in \{\text{True}, \text{False}\}$ , bool set
  - ▶  $(a \longrightarrow b) \in \{(a \longrightarrow b), x\}$ , bool set
  - ▶  $\text{"Hello"} \in \{\text{"Hello"}, \text{"World"}\}$ , string set
  - ▶  $\text{apple} \in \{\text{apple}, \text{banana}\}$ ,  $'a$  set,  
 $\text{apple}$  and  $\text{banana}$  are of arbitrary type
- ▶ value  $\{\text{True}, \text{False}\} :: \text{bool set}$

# Functional Programming

## Demo: Introduction.thy

```

fun find_fives :: "nat list  $\Rightarrow$  nat list" where
  "find_fives [] = []"
| "find_fives (x#xs) =
    (if x = 5 then 5#find_fives xs else find_fives xs)"
    
```

```

value "find_fives [1,3,5,8,5,2,5]"
    
```

```

lemma "find_fives [1,3,5,8,5,2,5] = [5,5,5]" by simp
    
```

Library function filter

```

filter :: "('a  $\Rightarrow$  bool)  $\Rightarrow$  'a list  $\Rightarrow$  'a list"
    
```

```

lemma "find_fives l = filter ( $\lambda$ x. x = 5) l"
  by(induction l, simp_all)
    
```

## Total Functions

- ▶ A *total* function is function that is defined for all input values
  - ▶ For computer scientists:

A total function must always terminate!

- ▶ Written ' $a \Rightarrow b$ '
- ▶ HOL is a total logic!
- ▶ Why total functions?
- ▶ Assume  $f$  of type  $nat \Rightarrow nat$ .  
 $f\ n = \text{while}(\text{true})\{\} \text{ return } 0$

1  $f\ n =$  does – not – terminate

2 does – not – terminate = does – not – terminate + 1

3 from (1) and (2):  $f\ n = (f\ n) + 1$

4 subtracting  $f\ n$  on both sides:  $0 = 0 + 1$

5  $0 = 1$

- ▶ HOL is total and every function needs a proof for that!

## Total Functions

Usually, the *total*-proof is for free.

```

7
8 fun find_fives :: "nat list => nat list" where
9   "find_fives [] = []"
10  | "find_fives (x#xs) = (if x = 5 then 5 # find_fives xs else find_fives xs)"
11
12 lemma "find_fives l = filter (λx. x = 5) l"
13 by(induction l, simp_all)
    
```

Auto update    Update    Detach    100%

Found termination order: "list\_size size <math>\lambda x.> \{>"

Here, the size of the list is used for the automatic proof.

- 1 The function obviously terminates for the empty list
- 2 If the list consists of  $x :: xs$ , all subsequent calls to *find\_fives* only get *xs*.

## Wrap-Up

- ▶ HOL = Functional Programming + Logic
- ▶ Strong Typing
- ▶ Total functions
- ▶ Proofs!



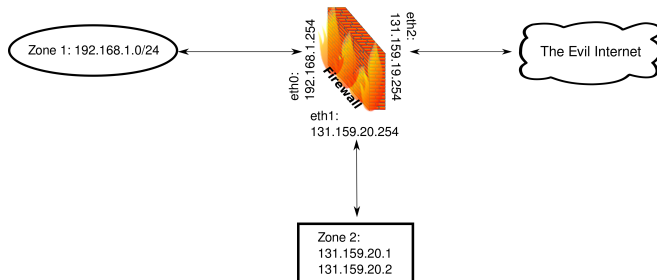
# Modeling Firewalls



## Firewalls – Introduction

- ▶ Access control on the network level
- ▶ Often positioned between a trusted and a less trusted network
  - ▶ E.g. between internal network and Internet

Example:



## Firewalls

- ▶ Decides per packet what to do
  - ▶ Allow – Forward the packet
  - ▶ Deny – Drop the packet
- ▶ A firewall is configured by a ruleset. Example:

#	Src IP	Dst IP	Proto	Src Port	Dst Port	Action
A	Alice	Gabby	TCP	*	80	Allow
B	Alice	8.8.8.8	UDP	*	53	Allow
C	*	*	*	*	*	Deny

Textual explanation on next slide (walls of text)

# Firewalls

- ▶ Rules are processed consecutively for every packet  
Assume a packet arrives
  - 1** Is the source Alice and the destination Gabby and the protocol TCP and the destination port 80 (HTTP)?  
If yes, the firewall allows the packet.  
If not, the next rule is processed.
  - 2** Is the source Alice and the destination 8.8.8.8 (google DNS server) and the protocol UDP and the destination port 53 (DNS)?  
If yes, the firewall allows the packet.  
If not, the next rule is processed.
  - 3** The firewall discards the packet.  
This default strategy is called blacklisting. Everything which is not explicitly allowed is denied.

#	Src IP	Dst IP	Proto	Src Port	Dst Port	Action
A	Alice	Gabby	TCP	*	80	Allow
B	Alice	8.8.8.8	UDP	*	53	Allow
C	*	*	*	*	*	Deny

# Firewalls

- ▶ For simplicity, our example ignores
  - ▶ Stateful firewalls
  - ▶ Answer packets.
    - Alice can send out HTTP and DNS request, but the firewall will never allow the answer to reach Alice.
  
- ▶ We only focus on the packet header.

#	Src IP	Dst IP	Proto	Src Port	Dst Port	Action
A	Alice	Gabby	TCP	*	80	Allow
B	Alice	8.8.8.8	UDP	*	53	Allow
C	*	*	*	*	*	Deny

# Modeling Firewalls – Representing Rules

Demo: `firewall.thy`

Step 1: Rules Basics

- ▶ Per Rule Action

- ▶ Allow
- ▶ Deny
- ▶ Pass — Rule does not apply, apply next rule

```
datatype action = Allow | Deny | Pass
```

- ▶ The Type of a Rule

- ▶ A rule processes an packet of arbitrary type 'p
- ▶ A rule defines an action for that packet
- ▶ A rule is a total function from packets to actions

```
type_synonym 'p rule = 'p  $\Rightarrow$  action
```

## Representing Rules – Example 1

- ▶ Let a packet consist of a tuple of arbitrary type  $'a \times 'b$
- ▶ Let  $'a$  be the source and  $'b$  be the destination address
- ▶ We can write a deny-all rule as
$$\lambda (src, dst). \text{ Deny}$$
- ▶ The type is  $('a \times 'b)$  rule

## Representing Rules – Example 2

- ▶ Let a packet consist of a tuple strings  $\text{string} \times \text{string}$
- ▶ The first entry is the source and the second the destination address
- ▶ We can write a rule which allows packets from *Alice* to *Bob* and denies all others

```

λ (src, dst). if src = ''Alice'' ∧ dst = ''Bob'' then
  Allow else Deny
    
```

- ▶ The type is  $(\text{string} \times \text{string})$  rule
  - ▶ For a packet  $(\text{'Alice'}, \text{'Carl'})$
  - ▶ We can show:
 

```

(λ (src, dst). if src = ''Alice'' ∧ dst = ''Bob'' then
  Allow else Deny) (''Alice'', ''Carl'') = Deny
            
```
- Applying the packet  $(\text{'Alice'}, \text{'Carl'})$  to the rule results in Deny

## Modeling Firewalls – Representing Rules

### Step 2: A Firewalls Rule Set

Probably, rule list would be a better terminology, we will soon prove why.

- ▶ The firewall has a ‘table’ of rules
- ▶ For packets of type 'p, we develop datatype 'p ruleset



## Modeling Firewalls – Representing Rules

datatype 'p ruleset

- ▶ Rules apply sequentially
- ▶ If no rule matches, a default rule must apply
- ▶ Default rules
  - ▶ DefaultAllow – Blacklisting  
Everything not prohibited is allowed by default
  - ▶ DefaultDeny – Whitelisting  
Everything not allowed is denied by default
- ▶ A Rule consists of a rule and its successor rules
  - ▶ Rule “'p rule” “'p ruleset”

```
datatype 'p ruleset = DefaultAllow
                    | DefaultDeny
                    | Rule “'p rule” “'p ruleset”
```

## Rulesets – Example 1

- ▶ Let a packet consist of arbitrary type 'a
- ▶ We can write a rule set with a Allow-All rule, a Deny-All rule, and a default allow strategy

```
Rule ( $\lambda p.$  Allow) (Rule ( $\lambda p.$  Deny) DefaultAllow)
```

- ▶ The type is 'p ruleset

## Rule Notation

- ▶ Instead of `Rule`, we will use the infix operator `§`
- ▶ Example:
  - ▶ `Rule x y` is written as `x § y`

Previous slide's example

`Rule ( $\lambda p.$  Allow) (Rule ( $\lambda p.$  Deny) DefaultAllow)`

Is written as

`( $\lambda p.$  Allow) § ( $\lambda p.$  Deny) § DefaultAllow`

## Conclusion

- ▶ We defined the syntax of
  - ▶ action
  - ▶ 'p rule
  - ▶ 'p ruleset
- ▶ Next, we will develop an example of a concrete ruleset for a concrete 'p
- ▶ Afterwards, the semantics of firewalls are specified.
  - ▶ Syntax: How to write things down
  - ▶ Semantic: Meaning

# Modeling Firewalls – Syntax Example

Demo: `firewall.thy`

## Semantics

We define a function `firewall` to capture the semantics of a firewall.

Step 1) The type

- ▶ A firewall operates according to a `ruleset`
- ▶ A firewall processes packets accordingly
- ▶ The firewall decides what to do with the packet

It returns an action

Hence, the type is `'p ruleset ⇒ 'p ⇒ action`

## Semantics of firewall

- ▶ The semantics are specified via pattern matching on the ruleset
  - ▶ an underscore ‘\_’ matches everything
- ▶ If the ruleset is only `DefaultAllow`, the firewall allows any packet  
`“firewall DefaultAllow _ = Allow”`
- ▶ If the ruleset is only `DefaultDeny`, the firewall denies any packet  
`“firewall DefaultDeny _ = Deny”`
- ▶ Most of the time, the ruleset will consists of a current `rule` and  
 and some `next_rules`  
`rule § next_rules`
- ▶ `next_rules` can be any ‘p’ ruleset, either further rules or one of  
 the default rules

## Semantics of `firewall`

`firewall (rule § next_rules) p`

- ▶ For a packet `p`
- ▶ The firewall examines `rule` and tests whether the rule matches `rule p`
- ▶ If the action is `Allow` or `Deny`, it applies the respective action
- ▶ If the action is `Pass`, the `next_rules` must be applied to `p`  
This is a recursive call to `firewall`



## Example

Demo: `firewall.thy`

## Termination of `firewall`

- ▶ The default cases trivially terminate
- ▶ For the case `rule § next_rules`, the size of the ruleset is decreased with every recursive call.  
If `firewall (rule § next_rules) p` is called
  - ▶ it either terminates with `Allow` or `Deny`
  - ▶ or calls `firewall next_rules p`
- ▶ This is an inductive argument.

# Add-In: Induction

## Induction

- ▶ Induction on natural numbers  $\mathbb{N}$
- ▶ To prove  $P\ n$  for  $n \in \mathbb{N}$
- ▶ Show that  $P$  holds for 0
- ▶ Assume  $P$  holds for  $n$  and show that it also holds for  $n + 1$

Induction rule in Isabelle/HOL (simplified)

$$P\ 0 \implies (\bigwedge n. P\ n \implies P\ (n + 1)) \implies P\ x$$

Read as follows

- ▶ To show  $P\ x$  (the goal right)
- ▶ Show  $P\ 0$
- ▶ and show  $\bigwedge n. P\ n \implies P\ (n + 1)$

Demo: [intro\\_induction.thy](#)

## Induction – Induction Rules

- ▶ Where do we get `nat.induct` from?
- ▶ Isabelle/HOL proves a lot automatically for us in the background
- ▶ How are natural numbers modeled?
  - ▶ Zero is a natural number
  - ▶ The successor `Suc` of a natural number is a natural number

```
datatype nat = Zero | Suc nat
```

```
8 datatype nat = Zero | Suc nat
```

The screenshot shows the definition of the natural number datatype and the list of proof obligations generated by Isabelle/HOL:

```

Proofs for inductive predicate(s) "nat_rep_set"
  Proving monotonicity ...
  Proving the introduction rules ...
  Proving the induction rule ...
Proofs for inductive predicate(s) "nat_rec_set"
  Proving monotonicity ...
  Proving the introduction rules ...
  Proving the elimination rules ...
  Proving the simplification rules ...
    
```

## Induction – Induction Rules

`datatype nat = Zero | Suc nat`

`nat.induct`

$P \text{ Zero} \implies (\bigwedge n. P n \implies P (\text{Suc } n)) \implies P x$

- ▶ Base case:  $P \text{ Zero}$
- ▶ Induction step:  $(\bigwedge n. P n \implies P (\text{Suc } n))$

## Induction – Firewall Rules

- ▶ Firewall rules are modeled similarly
  - ▶ DefaultAllow is a ruleset
  - ▶ DefaultDeny is a ruleset
  - ▶ A Rule consists of a rule and a ruleset

```
datatype 'p ruleset = DefaultAllow
                  | DefaultDeny
                  | Rule "'p rule" "'p ruleset"
```

- ▶ Two base cases
  - ▶ DefaultAllow
  - ▶ DefaultDeny
- ▶ One induction step
  - ▶ Assume it holds for some rules  $:: 'p$  ruleset.  
For any rule  $r :: 'p$  rule, show that it holds for Rule  $r$  rules.

## ruleset induction rule

thm ruleset.induct

$$\begin{aligned}
 & P \text{ DefaultAllow} \implies P \text{ DefaultDeny} \implies \\
 & (\bigwedge r \text{ rules. } P \text{ rules} \implies P (r \text{ \S rules})) \implies P \text{ ruleset}
 \end{aligned}$$

For an arbitrary ruleset, to prove  $P$  ruleset

- ▶ Prove that  $P$  holds for the base cases
  - ▶  $P \text{ DefaultAllow}$
  - ▶  $P \text{ DefaultDeny}$
- ▶ And the induction step
  - ▶ Assume  $P \text{ rules}$
  - ▶ Show  $P (r \text{ \S rules})$
  - ▶  $(\bigwedge r \text{ rules. } P \text{ rules} \implies P (r \text{ \S rules}))$



# Back to firewall

## Analyzing firewall

- ▶ For a ruleset  $r$  and a packet  $p$
- ▶ `firewall r p` never returns `Pass`
- ▶ It either returns `Allow` or `Deny`

lemma ‘‘`firewall r p = Allow  $\vee$  firewall r p = Deny`’’

proof(induction r)

...

Demo: `firewall.thy`

Yes, the `*.thy` is important!

## Analyzing Rule Sets

- ▶ What does the policy `allow_DNS § allow_HTTP § DefaultDeny` allow?
- ▶ Only UDP port 53 or TCP port 80
- ▶ Phrasing as a lemma
  - ▶ A firewall with this rule set
  - ▶ A packet `p` applied to it returns `Allow` if and only if
  - ▶ `p` is UDP port 53 or `p` is TCP port 80

```
lemma “firewall (allow_DNS § allow_HTTP § DefaultDeny) p = Allow
```

$\longleftrightarrow$

`(proto p = UDP ∧ dst_port p = 53) ∨`

`(proto p = TCP ∧ dst_port p = 80)”`

```
by(simp add: allow_DNS_def allow_HTTP_def)
```

- ▶ Why did the previous lemma require a (difficult) induction whereas this lemma could be directly solved by the simplifier?
- ▶ The previous lemma argued about arbitrary rule sets whereas this lemma had a known rule set.
- ▶ The `firewall` is defined in terms of the rule set.
- ▶ For unknown rule sets we (often) need induction over the rule set.
- ▶ If the rule set and the packet are given, the proof is often trivial as `firewall` is executable.

## Analyzing Rule Sets

- ▶ Rule sets may be inefficient or misleading
- ▶ For example, rules can *shadow* each other

```
deny_UDP § allow_DNS § allow_HTTP § DefaultDeny
```

- ▶ The rule `allow_DNS` is shadowed by `deny_UDP`
- ▶ It can never apply
- ▶ We can simplify this ruleset and preserve the semantics!

```
deny_UDP § allow_HTTP § DefaultDeny
```

- ▶ We can prove that the two rulesets are semantically equivalent if `firewall` is equal for both of them
- ▶ lemma ‘‘`firewall rules1 = firewall rules2`’’



## When are two functions equal?

- ▶ Two (total) functions  $f$  and  $g$  are equal iff they return the same result for every input

$$(f = g) = (\forall x. f\ x = g\ x) \quad \text{fun\_eq\_iff}$$

### Back to firewalls

- ▶ Two firewall rule sets `rules1` and `rules2` are semantically equivalent iff `firewall` is equal for them
- ▶ `firewall rules1` is equal to `firewall rules2` iff the two functions are equal for all packets

- ▶ The previous rule sets are semantically equivalent

$$\text{deny\_UDP} \S \text{allow\_DNS} \S \text{allow\_HTTP} \S \text{DefaultDeny}$$
$$\text{deny\_UDP} \S \text{allow\_HTTP} \S \text{DefaultDeny}$$

- ▶ This can be proven by the simplifier once it is rewritten to

$$\forall p. \text{firewall } ?r1 \ p = \text{firewall } ?r2 \ p$$

- ▶ This rule set can even be reordered

$$\text{allow\_HTTP} \S \text{deny\_UDP} \S \text{DefaultDeny}$$

## Ruleset Reordering

- ▶ We say a rule applies if it returns an action which is not Pass
- ▶ Rules  $r_1$  and  $r_2$  can be reordered if there is no packet such that both rules apply to the packet simultaneously

$$\nexists p. r_1 p \neq \text{Pass} \wedge r_2 p \neq \text{Pass}$$

- ▶ This is equal to the following:
- ▶ For all packets, at least one of  $r_1$  and  $r_2$  must not apply

$$\forall p. r_1 p = \text{Pass} \vee r_2 p = \text{Pass}$$

- ▶ This condition is sufficient to allow reordering of the first rules  
`firewall (r1 § r2 § r3) = firewall (r2 § r1 § r3)`
- ▶ The condition is not necessary (try quickcheck)



# Rule Sets or Rule Lists?

## Rule Sets or Rule Lists

- ▶ The rulesets we saw bear great resemblance to lists
- ▶  $r_1 \text{ \& } r_2 \text{ \& } r_3 \text{ \& } r_4 \dots$
- ▶ The order of the elements is important
- ▶ Can we represent firewall rules as list?

## Lists

- ▶ `[1 :: nat, 2, 3, 5] :: nat list`
- ▶ `recall find_fives`
- ▶ `rev [1 :: int, 2, 3, 4] = [4, 3, 2, 1]`
- ▶ lemma “`rev (rev l) = l`” by simp
- ▶ `[a, b, c] :: 'a list`
- ▶ `[allow_DNS, allow_HTTP] :: packet rule list`

## Semantics of a firewall with a rule list

- ▶ We define the semantics of a firewall whose rules are defined as list

Type: `'p rule list`  $\Rightarrow$  `'p`  $\Rightarrow$  `action`

- ▶ The list is processed sequentially – the rule in the list which matches first is applied
- ▶ If a rule returns `Pass`, the next list item is observed
- ▶ We use Default-Deny semantics – the empty list corresponds to `Deny`
- ▶ Note the similarity
  - ▶ `ruleset: rule § next_rules`
  - ▶ `rule list: rule # next_rules`

See `firewall_list_defaultDeny`

## ruleset **VS.** rule list

- ▶ How does the ruleset and rule list correspond?
- ▶ we can translate them  
`'p ruleset  $\Rightarrow$  'p rule list`
- ▶ With the Default-Deny semantics
  - ▶ `DefaultDeny` corresponds to the empty list `[]`
  - ▶ `DefaultAllow` corresponds to the one-element list with an allow-all rule `[( $\lambda p.$  Allow)]`
  - ▶ the correspondence rule `§ next_rules` and `rule # next_rules` is defined recursively

See `ruleset_to_rulelist`

## ruleset to rule list – Example

- ▶ A ruleset can be translated to a rule list

```
ruleset_to_rulelist (allow_DNS § allow_HTTP § DefaultDeny)
                    = [allow_DNS, allow_HTTP]
```

- ▶ Does this preserve the semantics?
- ▶ in this example, yes!

```
firewall (allow_DNS § allow_HTTP § DefaultDeny) =
firewall_list_defaultDeny [allow_DNS, allow_HTTP]
```

- ▶ Does this hold in general?

## Relating the semantics

- ▶ Translating a ruleset to a rule list, the `firewall` and the `firewall_list_defaultDeny` are equal

```
lemma fw_eq_fwlist:  ‘‘firewall r =  
firewall_list_defaultDeny (ruleset_to_rulelist r)’’
```

- ▶ The proof (idea)
  - ▶ First, use `fun_eq_iff`
  - ▶ This leaves the subgoal that the two definitions are equal for all packets
  - ▶ Then, apply induction over the ruleset `r`

Demo: `firewall.thy`

Yes, the `*.thy` is important!

# Conclusion



## Conclusion

- ▶ Basics of modeling in Isabelle/HOL
  - ▶ FOL, HOL,  $\lambda$ -calculus, polymorphic types
  - ▶ Types and total functions
  - ▶ Induction
- ▶ Firewalls with `rulesets`
  - ▶ Syntax
    - ▶ actions, rules, `ruleset`
  - ▶ Semantics
    - ▶ Proofs about the semantics
    - ▶ Termination, never returns `Pass`, ruleset shadowing, ruleset reordering
- ▶ Firewall modeled with lists
- ▶ Semantical equality

# Thanks for your attention!

Questions?