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Tutorials for Network Coding (IN3300)
Tutorial 4 – 2014/11/20

Problem 1 Lossy wireless networks

We consider the four-node wireless relay network $G = (N, H)$ depicted in Figure 1 in the lossy hypergraph model with orthogonal MAC. The solution of most subproblems can be written as table (see pre-printed Table 1).

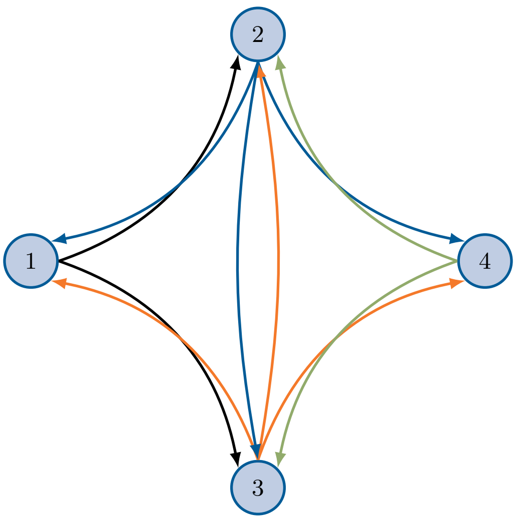

Figure 1: Four-node relay network

a)* Explicitly state the set of hyperarcs H.

b) Number the hyperarcs $(a, B) \in H$ in lexicographic ascending order, i.e., $(a, B) < (a', B')$ if

1. $a < a'$ *or*
2. $a = a' \;\wedge\; |B| < |B'|$ *or*
3. $a = a' \;\wedge\; |B| = |B'| \;\wedge \min B < \min B'$,

such that $j \equiv (a, B)$ with $j \in \{1, 2, \ldots\}$ for all $(a, B) \in H$.

c)* Explicitly state all arcs $(a, b) \in A$ that are induced by each of the hyperarcs $(a, B) \in H$.

d) Draw the graph $G' = (N, A)$ that is induced by G.

e) Number the arcs $(a, b) \in A$ in lexicographic ascending order, i.e., $(a, b) < (a', b')$ if

1. $a < a'$ *or*
2. $a = a' \wedge b < b'$,

such that $k \equiv (a, b)$ with $k \in \{1, 2, \dots\}$ for all $(a, b) \in A$. Also state by which hyperarc $j \equiv (a, B) \in H$ a given arc $k \equiv (a, b) \in A$ is induced by.

f) Enumerate the sets A_j for all $j \equiv (a, B) \in H$ such that $(a, b) \equiv k \in A_j$ if hyperarc j induces arch k.

g) State the hyperarc-arc incidence matrix $\boldsymbol{N}$.

h) State the hyperarc-hyperarc incidence matrix $\boldsymbol{Q}$.

We now consider a bidirectionally coded session between nodes 1 and 4. Assume that each arch $k \in A$ has unit capacity and a link error probability of $0 \leq \epsilon_k \leq 1$.

i) Determine the hyperarc capacity region $\mathcal{Z}$ assuming that

$$\begin{aligned} \tau_1 = \tau_4 &= \tau, \\ \tau_2 = \tau_3 &= \theta, \\ \epsilon_{13} = \epsilon_{31} = \epsilon_{24} = \epsilon_{42} &= \xi, \\ \epsilon_{12} = \epsilon_{21} = \epsilon_{34} = \epsilon_{43} &= 0, \text{ and} \\ \epsilon_{23} = \epsilon_{32} &= \delta. \end{aligned}$$

j) Determine the broadcast capacity region $\mathcal{Y}$.

k) Enumerate all s–t cuts S and their capacities $v(S_i)$.

l) Which cuts are redundant, i.e., which cut can not be the min cut?

m) Find the maximum bidirectional communication rate $r = \min(r_1, r_4)$ assuming that $\theta = \frac{1}{2} - \tau$ by computing the min-cut value.

n) Determine τ such that r is maximized.

o) Discuss the extreme cases $\xi \in \{0, 1\}$ and $\delta \in \{0, 1\}$.

$(a, B) \in H$	$j \equiv (a, B)$	(a, b)	A_j	z_j	y_j

Table 1: Fill in values from different subproblems.