Problem 1  FEC with ARQ

Consider a simple wireless network consisting of two nodes $s$ and $t$. Node $s$ transmits packets of $l = 15\,808$ bit each. The channel has a bit error rate of $\epsilon = 10^{-4}$.

If a transmission of $s$ is successfully received by $t$, an acknowledgement is triggered and sent back to $s$. We assume orthogonal scheduling, i.e., there are no additional losses due to collisions. Further we assume that acknowledgements do not get lost.

a) Let $X$ be a random variable that counts the number of bit errors in a given packet. Determine the probability for a successful transmission, i.e., $\Pr \{X = 0\}$.

b) Let $T$ denote a random variable that counts the number of transmissions until a packet is acknowledged. Determine $\Pr \{T = i\}$ and $\Pr \{T \leq i\}$ in general and for $i = 7$.

c) Determine the expectation $E[T]$, i.e., the average number of transmissions that are needed until successful reception.

To secure transmissions node $s$ now employs a FEC code which maps source symbols of $k = 247$ bit to coded symbols of $n = 255$ bit. The code is able to detect and correct a single bit-error in each coded symbol.

d) Determine the probability that a single block can be recovered at the receiver.

e) Let $Z$ count the number of incorrect transmitted symbols. Determine the probability for a successful transmission during the first attempt for the whole packet if FEC is used.

Problem 2  Linear dependency of random vectors

Let $c \in F_q^n[x]$ denote coding vectors which are drawn independently and uniformly distributed. Coding vectors are assembled to a coding matrix $C = [c_1 \ldots c_m] \in F_q^{n \times m}[x]$ at the receiver. The receiver is able to decode if $\text{rank} \ C = n$. Let $\rho_{mn}^k$ denote the probability that $\text{rank} \ C = k \leq n$ after receiving $m \geq k$ coding vectors.

a)* Determine the probability $\rho_{1n}^1$, i.e., the probability to draw a random vector $c \neq 0$.

b) Determine the probability $\rho_{2n}^2$, i.e., two random vectors are linear independent.
c) Determine the probability $\rho_{mn}^m$ for $m \leq n$.

d) Determine the probability $\rho_{mn}^n$ for $m \geq n$.

Let $X$ denote a random variable counting the number of vectors needed to span $F_q^n[x]$. The probability for $X < n$ is obviously 0. For $m > n$ the probability is given by $\rho_{m-1,n}^n$ and thus

$$
\Pr[X < m] = \begin{cases} 
0 & m \leq n, \\
\rho_{m-1,n}^n & m > n.
\end{cases}
$$  

(1)

e)* Derive $E[X]$ for $n = 32$ and $q \in \{2, 8, 16, 32\}$. As far as we know $E[X]$ has no closed form. So simplify as much as possible and then use Matlab to determine numerical results.

**Hint:** $E[X] = \sum_{m=1}^{\infty} \Pr[X \geq m]$. 