Looping with Untyped λ-calculus in Python & Go

Lambda calculus is an important formal system used in theoretical computer science to describe computation.

The Y combinator introduces recursion into this language and is defined as $\lambda f.\ (\lambda x.\ f\ (x(x)))\ (\lambda x.\ f\ (x(x)))$. In this one-pager, we are going to practically derive some of its core ideas. We will use our favorite untyped λ-calculus shell, which is `ipython3`. Let's get started.

```
user@box:~$ ipython3
In [1]:
```

The rules of λ-calculus only allow the following:

1. Referencing bound variables: given x, we may write x.
2. Defining anonymous functions: given e, we may write $\lambda x.\ e$. Formally, this is called lambda abstraction.
3. Calling functions: given e and x, we may write $e(x)$. Formally, this is called function application.

This is all we need to describe any computation. We won't need control flow statements, such as **`if`**, **`while`**, or **`for`**. We won't define variables and won't **`def`**ine non-anonymous functions. Of course, `import os; os.system("python -c'...'")` and `eval` are prohibited. For convenience, we allow ourselves a bit of arithmetic, namely the `+` function.

We will only use **`lambda`** and `+` to build our own infinite loop. Our goal is to `print` all natural numbers. We want to call `print(n)` for all `n`, til the physical limits of our underlying finite machine (python's recursion depth) stop us.

Since the `print` function is given, we reference it (rule 1).

```
In [1]: print(n)
NameError: name 'n' is not defined
```

Since `n` was not given, we get an error. To make `n` available in this scope, we build a lambda abstraction (rule 2).

```
In [2]: lambda n: print(n)
In [2]: <function __main__.<lambda>>
```

We get a valid function. To test it, we apply the function (rule 3) to our starting value, which gives the expected result.

```
In [3]: (lambda n: print(n))(1)
1
```

Now, we only need to print the remaining natural numbers. The following recursive function[1] would solve our problem: **`def`** `f(n): print(n)+f(n+1)`. Yet, the rules only permit to define anonymous functions. We continue with a trick from mathematics. We just assume stuff! We assume `f` already exists and also assume `f` references our current function.

```
In [4]: lambda n: print(n)+f(n+1)
In [4]: <function __main__.<lambda>>
```

Let's test.

```
In [5]: (lambda n: print(n)+f(n+1))(1)
1
NameError: name 'f' is not defined
```

[1] Why can we combine `print` and `f` with the `+` operator? The function `print` returns **`None`** and `+` is not defined on **`None`**. We don't see the expected `TypeError: unsupported operand type(s) for +`, since `f` never returns. The cool kids say that `f` diverges.

There is no magic `f` in our scope. Since we don't know `f`, let's assume someone will provide it for us.

```
In [6]: lambda f, n: print(n)+f(n+1)
In [6]: <function __main__.<lambda>>
```

Since `f` needs to refers to ourselves, we need to pass ourselves along when calling ourselves recursively.

```
In [7]: lambda f, n: print(n)+f(f,n+1)
In [7]: <function __main__.<lambda>>
```

Looks good, we just need to provide the function `f` and the starting value `1`. Let's mock `f` temporarily by `...`.

```
In [8]: (lambda f, n: print(n)+f(f,n+1))(..., 1)
1
TypeError: 'ellipsis' object is not callable
```

Works as expected, we print `1` and try to call `...` afterwards. Now we need a real implementation for `f` instead of `...`. Our `f` should be the function we are currently implementing. Copy and paste to the rescue!

```
In [9]: (lambda f, n: print(n)+f(f,n+1))(
   ...:  lambda f, n: print(n)+f(f,n+1), 1)
1
2
3
...
985
986
987
RecursionError: maximum recursion depth exceeded
while calling a Python object
```

That escalated quickly!

Goal achieved!

Debrief. As an exercise to the reader, simplify the previous expression such that it fits in a single line. The solution is below.

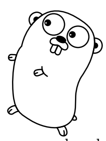
```
(lambda f: f(f,1))(lambda f, n: print(n)+f(f,n+1))
```

What is the type of `f`? Well, it's a function, where the first argument is a function, where the first argument is a function, where the first argument is a function, ..., and the second argument is a number.

We port our code to Golang – a statically typed language.

```go
package main
import "fmt"

func main() {
  func(f interface{}) {
    f.(func(interface{}, int))(f, 1)
    }(func(f interface{}, n int) {
      fmt.Println(n)
      f.(func(interface{}, int))(f, n+1)
  })
}
```



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In fact, whenever we write `interface{}`, it should be `func(func(func(..., int), int), int)`. But since Golang, as a statically typed language, does not permit infinite types, we use `interface{}`, which is a type synonym for `yolo`.

Cheers.