Technische Universität München
Lehrstuhl Informatik VIII
Prof. Dr.-Ing. Georg Carle
Prof. Dr.-Ing. Wolfgang Utschick
Stephan M. Günther
Maximilian Riemensberger

Tutorials for Network Coding (IN3300)

Tutorial 3 – 2014/11/18

Problem 1 Lossy wireless networks

We consider the three-node wireless relay network $G = (N, H)$ depicted in Figure 1 in the lossy hypergraph model with orthogonal MAC.

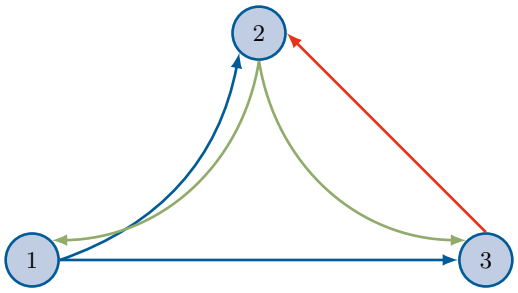

Figure 1: Three-node relay network

a)* Explicitly state the set of hyperarcs H.

b) Number the hyperarcs $(a, B) \in H$ in lexicographic ascending order, i.e., $(a, B) < (a', B')$ if

1. $a < a'$ *or*
2. $a = a' \ \wedge \ |B| < |B'|$ *or*
3. $a = a' \ \wedge \ |B| = |B'| \ \wedge \min B < \min B'$,

such that $j \equiv (a, B)$ with $j \in \{1, 2, \ldots\}$ for all $(a, B) \in H$.

c)* Explicitly state all arcs $(a, b) \in A$ that are induced by each of the hyperarcs $(a, B) \in H$.

d) Draw the graph $G' = (N, A)$ that is induced by G.

e) Number the arcs $(a, b) \in A$ in lexicographic ascending order, i.e., $(a, b) < (a', b')$ if

1. $a < a'$ *or*
2. $a = a' \ \wedge \ b < b'$,

such that $k \equiv (a, b)$ with $k \in \{1, 2, \dots\}$ for all $(a, b) \in A$. Also state by which hyperarc $j \equiv (a, B) \in H$ a given arc $k \equiv (a, b) \in A$ is induced by.

f) Enumerate the sets A_j for all $j \equiv (a, B) \in H$ such that $(a, b) \equiv k \in A_j$ if hyperarc j induces arch k.

g) State the hyperarc-arc incidence matrix $\boldsymbol{N}$.

h) State the incidence matrix $\boldsymbol{M}$ for G'.

i) State the hyperarc-hyperarc incidence matrix $\boldsymbol{Q}$.

Assume that each arch $k \in A$ has unit capacity and a link error probability of $0 \leq \epsilon_k \leq 1$.

j) Determine the hyperarc capacity region $\mathcal{Z}$.

k) Determine the broadcast capacity vector $\boldsymbol{y}$.

l) Explicitly state the lossy hyperarc flow bound.

m) Enumerate all $s - t$ cuts S and their respective capacities $v(S_i)$ for $s = 1$ and $t = 3$.

n) State the min-cut capacity r for a flow from s to t in dependency of τ_1 and τ_2.

o) Determine τ_1 and τ_2 such that r is maximized.

We now consider the multicast $s = 1$ and $T = \{2, 3\}$.

p) Determine the missing $s - T$ cut and its capacity.

q) State the optimization problem to maximize the multicast capacity r'.

r) Determine the maximum multicast rate r'^* by solving the problem.
Hint: It is sufficient to differentiate between cases and to express τ_2, τ_3 by means of τ_1. Except for the trivial case, the expression for τ_1 is not nice.