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Master Course Computer Networks

Homework 6
(submission until February 4 into INBOX located in front of 03.05.052)

Note: Subproblems marked by * can be solved without preceding results.

RTTs Delay distribution

In this problem we estimate the delay (RTT) distribution between two systems. First, we probe the RTT to the destination many times and save the results. Based on the data we generate an empirical distribution. Finally, we approximate the empircal distribution by a suitable PMF with appropriately chosen paramters. The outcome should be similar to the plot of delay distributions in the lecture slides.
It is essential that you conduct your measurements from a reasonable fast internet connection. In particular you must *not* use wireless connections. If you do the measurements from your private internet connection, you should avoid any other activity during measurement. If possible, you should also *not* use your VM for the measuremnts because the target is located in the same subnet as your VMs, i. e., you would get extremely low RTTs which might not resemble a path over multiple hops. However, you can try it if you like.

a)* Describe your test setup, i. e., from which machine are your performing your tests, which is the (assymmetric bandwidth) of the connection.

The following tests are performed from 83.133.105.60 which has a (shared) symmetric connection of 1 Gbit/s.

b)* Measure the RTT using ICMP echo requests to the target located at 188.95.234.9. You should do at least 5000 probes one after another (use ping with "-n" and "-i 0.01" or "-i 0.001", do not use preloading). Store the results in a text file for later processing.

c) Extract the RTTs from the resulting text file and calculate the minimum, maximum, mean, and median.

$$
\begin{aligned}
x_{\text{min}} &= 22.9\,\text{ms} \\
x_{\text{max}} &= 30.3\,\text{ms} \\
x_{\text{mean}} &= 24.0\,\text{ms} \\
x_{\text{median}} &= 23.3\,\text{ms}
\end{aligned}
$$

d) What is the minimum delay comprised of?

The minimum delay is comprised of many different terms such as

- propagation delay of signals,
- serialization delay of frames,
- queuing delays, and
- processing delays.

If the minimum delay would be solely comprised of signal propagation, then considering the minimum RTT of 22.9 ms and assuming a symmetric path would result in a distance of 2290 km between both nodes. Although this is most probably an overestimation, it limits the geographic range wherein 188.95.234.9 must be located.

e) Use a CAS (or some combination of awk/pgfplots) to create a histogram of the results. Given $N = 5000$ samples, a total of $\sqrt{N} \approx 71$ bins might be a meaningful choice.

See Figure 1. Note that the sum of the discrete values evaluate to 1.

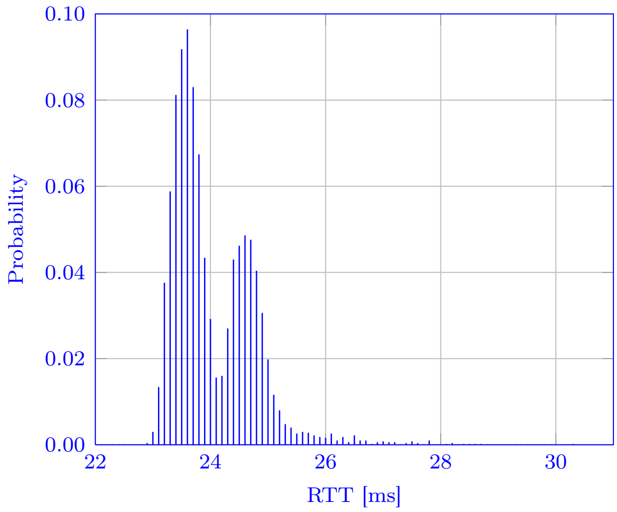

Figure 1: Empircal PMF / histogram

One possibility to estimate the distribution is to use a a shifted parameterized Rayleigh distribution. Its PMF is given by

$$f(x;\sigma) = \frac{x - x_{\min}}{\sigma^2} e^{-\frac{(x-x_{\min})^2}{2\sigma^2}}, \ \forall x \geq x_{\min}. \tag{1}$$

Since the minimum RTT measured (denoted by $x_{\min}$) is probably not 0, the Rayleigh distribution is shifted such that it starts at $x_{\min}$ instead of 0.
The parameter σ can be determined using the maximum likelihood estimator derived from the samples x_i and is given as

$$\sigma = \sqrt{\frac{1}{2N} \sum_{i=1}^{N} (x_i - x_{\min})^2}. \tag{2}$$

f)* Confirm Equation (2) (we do not expect you to determine MLEs in the exam).

The likelihood function is given as

$$L(\sigma \mid x_1, \ldots, x_N) = \prod_{i=1}^{N} f(x_i; \sigma).$$

As we seek to maximize $L(\sigma \mid x_1, \ldots, x_N)$ we can also maximize $\ln L(\sigma \mid x_1, \ldots, x_N)$ which gives

$$\begin{aligned}
\ln L(\sigma \mid x_1, \ldots, x_N) &= \sum_{i=1}^{N} (x - x_{\min}) - 2N \ln \sigma - \frac{1}{\sigma^2} \sum_{i=1}^{N} (x - x_{\min})^2 \\
\frac{\partial L}{\partial \sigma} &= -\frac{2N}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^{N} (x - x_{\min})^2 \\
\Rightarrow \sigma &= \sqrt{\frac{1}{2N} \sum_{i=1}^{N} (x_i - x_{\min})^2}
\end{aligned}$$

g) Plot the $f(x; \sigma)$ together with the empirical distribution.

See Figure 2. Note that the continuous PMF has been scaled by 10^{-1}.

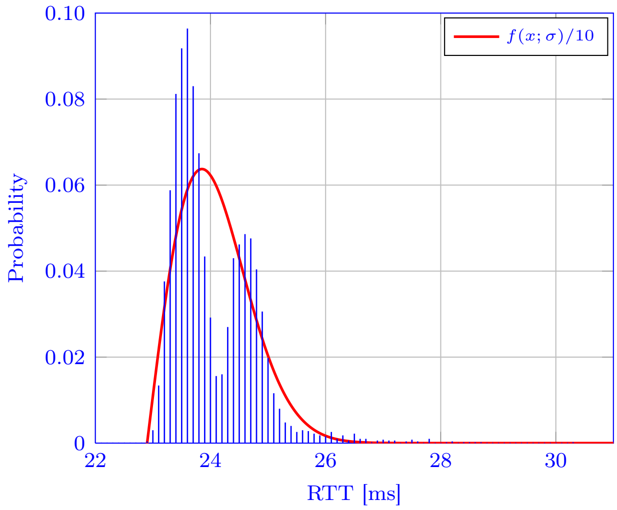

Figure 2: Estimated continuous PMF

h) Compare the emperical mean with the expectation of $f(x; \sigma)$.

$\mathbb{E}[f(x; \sigma)] = x_{\min} + \sigma\sqrt{\pi/2} \approx 24.1$, which is only 0.4 % off the empircal value.