# Network Security

## Random Numbers

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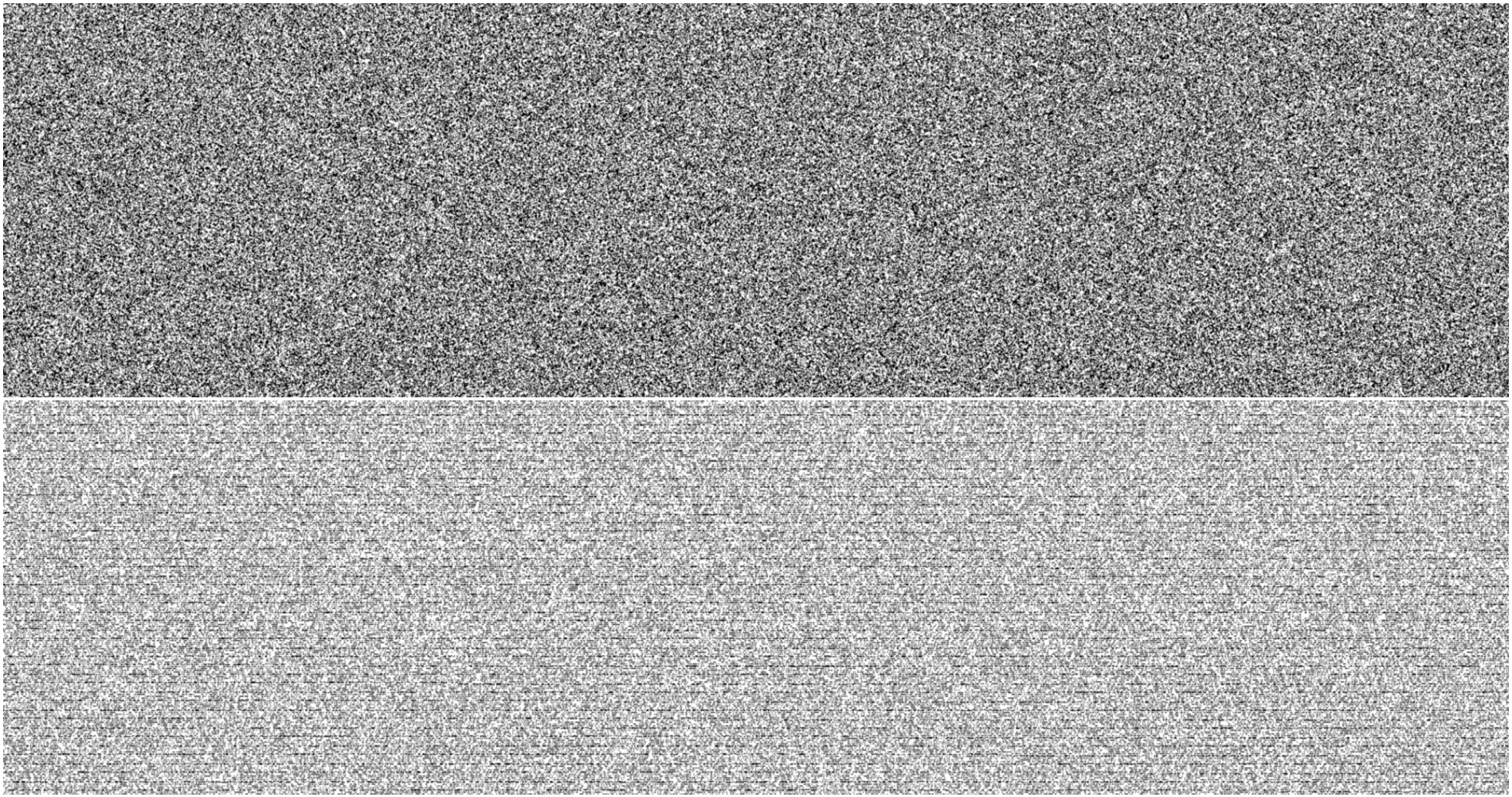
# What does ‘Random’ mean?



Random noise in your browser: Safari (top); V8 (bottom).

# Entropy

- "randomness" can be described by unpredictability
- A measure for "unpredictability" is "entropy"
- Let $X$ be a random variable which outputs a sequence of $n$ bits
- The Shannon information entropy is defined by:

$$H(X) = -\sum_{x} P(X = x)\ln_2(P(X = x))$$

- Entropy is maximized for a uniform distribution
  - I.e. every Bit is equally likely
  - Def.: truly random
- In this case: $H(X) = n$

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- Assume the attackers knows the passwors is printable
- Entropy is about 6.66

# Collecting Entropy

- Hardware-based; physical phenomena
  - time between emission of particles during radioactive decay
  - thermal noise from a semiconductor diode or resistor
  - frequency instability of a free running oscillator
  - the amount a metal insulator semiconductor capacitor is charged during a fixed period of time
  - noise of microphone or camera
- Software-based
  - the system clock
  - elapsed time between keystrokes or mouse movement
  - buffers
  - user input
  - OS stats, e.g. network load
- Attacker must not be able to guess/influence the collected values

# Cheap Randomness

- Getting entropy is expensive
- Pseudo-Random Number Generator (PRNG):
  - Deterministic algorithm
  - Input: truly random binary sequence of length, seed
  - Output: sequence of random-looking numbers
- seed: small amount of initial entropy

# PRNG – Example

- linear congruential generator

$$y_i = a \cdot y_{i-1} + b \ \ MOD\ q$$

- predictable $\rightarrow$ not cryptographic!

# Cryptographically Secure Pseudo Random Number Generator – CSPRNG

- The length of the seed should be large enough to make brute-force search over all seeds infeasible
- The output should be indistinguishable from truly random sequences
  - no polynomial-time algorithm can correctly distinguish between an output sequence of the generator and a truly random sequence
- The output should be unpredictable for an attacker with limited resources, without knowledge of the seed

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